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ABSTRACT

This document contains a list of recommended essential mathematical skills and outcomes developed for Arizona students from primary through secondary grades. These skills have been developed by using a spiral structure of individual clusters of objectives that are taught at an initial level and then retaught at higher levels as students progress through each grade. This list was based on a foundation statement which said that students deserve an opportunity to participate in a curriculum which fosters their understanding of mathematics, encourages mathematical thinking, provides for career options, and nurtures an interest in the field. The guide discusses the following aspects of mathematics: problem solving; concrete learning experiences; the learning environment; teaching for understanding; the nature of mathematics and how it should be taught; the significance of mathematics; student attitudes; addressing the needs of a diverse student population; technology; and staff development. (KR)

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ARIZONA ESSENTIAL SKILLS FOR MATHEMATICS

SE051700

ARIZONA DEPARTMENT OF EDUCATION
C. Diane Bishop, Superintendent
July 1987

ARIZONA ESSENTIAL SKILLS

FOR MATHEMATICS

ARIZONA DEPARTMENT OF EDUCATION

C. Diane Bishop, Superintendent

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FOREWORD

The definition of essential mathematical skills was not an easy task. From the outset, the Essential Skills Committee assigned to the task sought to avoid participating in a limitation of the essential skills to the traditional component of the "three Rs" because society has become too highly technological and complex. Evaluation is an important educational tool, but it is inappropriate to define essentials by what can presently be tested. A final overall concern was to avoid falling into a trap of defining a few "skills" when people use combinations of skills, knowledge and feelings in adjusting to the world around them. The Committee urges the State Board of Education to avoid concentration on training students to use a few skills at the expense of preparing students for the ever-changing society in which they must live. The temptation to concentrate solely upon easy-to-teach, easy-to-test information must be resisted. Mathematics education must focus not only on short-term objectives, but also on producing citizens who can make and form judgments and utilize rational problem-solving skills.

The Essential Skills documents which follow reflect the Committee's recommended Essential Skills for the mathematics education of Arizona students. State Board Rule R-7-2-301.01 declares that each student shall "demonstrate mastery as defined by the local governing board" of the Essential Skills. The Committee declined to define mastery and recommends that the State Board of Education and the Arizona Department of Education similarly resist the temptation to define this term.

The Committee agreed that mastery is a concept which should be defined by local districts. The Committee members, after careful consideration, firmly believed it was inappropriate for the Committee or the State Board of Education to deprive local districts of autonomy by setting a statewide performance level. The concern most expressed at the public hearings was that a statewide structure defining mastery would be imposed on local districts. The Committee shared this concern. The skills which follow are essential for Arizona students. The local districts, however, must define the performance level they consider to be mastery so that they control the variables of diverse student populations and community standards.

The Committee also strongly believed that the present emphasis on standardized testing is misplaced. One of the individuals at a regional hearing squarely addressed this issue:

> The teachers in Arizona can't serve two masters. If they want the teachers to do a good job of teaching math they can use the [Essential Skills] document which I have in my hands and throw out the ITBS tests or teach the ITBS tests and throw out the [Essential Skills] document which I hold.

The Committee did not go so far as to recommend total abandonment of standardized testing. Standardized testing can indicate proficiency on some of the Essential Skills, but many other skills do not lend themselves to measurement by current testing methods. The Committee was of the uniform belief that it is inappropriate to use standardized testing as the engine by which to drive mathematics education in this state.

These skills have been developed utilizing a spiral structure. This is a comprehensive structure which means individual clusters of objectives are taught at an initial level and then retaught at higher levels as students progress through each grade. Students need to begin the program at the primary level and continue through high school to achieve optimal results. It was not the

intent of this Committee to expect students to achieve a high level of proficiency in these skills without the benefit of this structure. Students will achieve proficiency in these skills once the skills have been in place long enough to provide continual exposure to them through the spiraling education process.

THE REVISION PROCESS

BACKGROUND

THE COMMITTEE

The Arizona State Board of Education appointed this advisory committee to perform the initial tasks involved in revision and review. One-third of the Committee members were appointed as professional educators. The remaining two-thirds of the Committee were citizen members although several had past and present experience as educators, many as former teachers or members of school boards or districts. Notwithstanding their extremely broad and diverse backgrounds, Committee members shared an intense interest in mathematics education.

The educator members of the Committee were from all sectors of education, including elementary, secondary and university teachers and administrators. A staff facilitator was appointed to the Committee by the Department of Education at the Board's request. This person was Kay Dean, Mathematics Specialist.

Other key resources at the Committee's disposal were mathematics curricula from several other states and some of the foreign countries which scored higher than the United States on the recent International Mathematics Test. Some of the approaches from the other states and countries were found suitable or adaptable for use in Arizona and were incorporated into the Committee's final report and the Essential Skills themselves.

The Mathematics Essential Skills Committee was chaired by Mr. James Nelson from Phoenix. Members of the Committee and communities they represented are

Chambers:	Ms. Elsie Duran	Scottsdale:	Mr. Larry Smith
Eagar:	Mr. Stan Smith	Tempe:	Dr. Ron Brown
			Ms. Jan Burkhardt
Flagstaff:	Dr. Charles Little		Ms. Joan Pfuhl
Mesa:	Ms. Bonnie Morales	Tucson:	Ms. Carol Brooks
	Dr. Steven Walters		Ms. Dale Curtis
			Ms. Ilona Gay
Phoenix:	Ms. Martha Baca		Dr. Virginia Horak
	Ms. Cindy Gilbert		
	Ms. Joy Hanley	Williams:	Mr. Dan Baertlein
	Ms. Joanne Kimura		
	Ms. Jan Miller		
	Mr. James Nelson		
	Ms. Adria Renke		
	Ms. Susan Saldaña		
	Mr. Dean Sulzer		

THE PROCESS

The Committee had its first meeting in May 1986. Subsequent to this initial meeting, the Committee met on a monthly basis through December 1986.

During the course of discussions, the Committee displayed a willingness to examine in great detail all facets of mathematics education. The Committee made a conscious decision early in the process, however, to keep the "big picture" in focus even while examining the details necessary for establishing the Essential Skills. Among the important considerations guiding the Committee were the following:

1. The overall objective of these Essential Skills is to increase student math proficiency. This objective can only be met by improving the quality of mathematics instruction.

2. The Essential Skills outlined here are intended as a flexible guide for the local districts. The Essential Skills should be adapted to local conditions so that the needs of all students in the educational community can be met.

3. The members of the Committee were uniform in their belief that districts and schools should be responsible for teaching these skills and striving to exceed them whenever possible.

4. Although the Essential Skills were designed primarily as a guide to local districts and individual schools in their development of curriculum and implementation of that curriculum, they also are intended to provide guidance to the textbook selection committee.

DISTRICT RESPONSES AND HEARINGS

The Committee was keenly aware that the success or failure of the proposed revisions to the Essential Skills in Mathematics were contingent, in large part, on the acceptance and implementation of those skills by local districts and schools. Therefore, a draft of the proposed revisions was submitted to school districts statewide for district-level input.

Superintendents were asked to respond to the revision or designate someone else in their district to do so. The individual districts were also provided a response sheet on which they could report their impressions, and were informed of the dates, places and times of the three regional public hearings.

Response sheets were received from many districts and schools within districts. Some districts and schools made copies of the entire document and teachers were asked to respond. All responses were reviewed by Committee members. Complete data, including a narrative of comments and statistical reports, can be obtained from the Department of Education. Some districts, schools and teachers wrote separate letters or attached separate comments, and these narrative responses are also available from the Department of Education.

Hearings were scheduled in three locations across the state to provide for direct citizen and district input.

Schedule of Public Hearings

Flagstaff, Arizona

Place:	Flagstaff High School Mini-Auditorium
Date:	Tuesday, January 13, 1987
Time:	7 p.m. – 9 p.m.

Phoenix, Arizona

Place:	Arizona Department of Education 1535 West Jefferson, Room 417
Date:	Wednesday, January 14, 1987
Time:	5 p.m. – 7 p.m.

Tucson, Arizona

Place:	Board Room, Tucson Unified School District No. 1 1010 East Tenth Street
Date:	Thursday, January 15, 1987
Time:	5 p.m. – 7 p.m.

Approximately 50 people attended the hearings, and participants' comments were recorded on notes and tape. A summary of the notes taken at the hearings is available from the Department of Education.

Comments made during the hearings were positive, and specific suggestions for improvement were made. Concerns expressed at all sites related to the degree of autonomy local districts would have in implementing the Essential Skills in Mathematics. Flagstaff participants were concerned that the revision was overly ambitious and ignored various problems which districts in the northern portion of the state faced. These problems included budget restrictions and high percentages of minority students. Flagstaff participants also expressed a concern that the State Board of Education, the Arizona Department of Education and the Arizona Legislature were burdening school districts with a revision when they had only recently managed to incorporate the 1984 list into their curriculum.

FINAL REVISION

The Committee met on February 3, 1987, to consider all responses to the draft document. Committee members made recommendations that the draft be further revised to accommodate many of the responses; therefore, several changes were made to the draft the districts reviewed. These revisions included deletion of some skills from the list in response to concerns that some of them were too sophisticated for the grade level. The final revision, as submitted for approval by the State Board of Education on February 23, 1987, reflects the Committee's decisions regarding responses received from districts, schools, and teachers across the state.

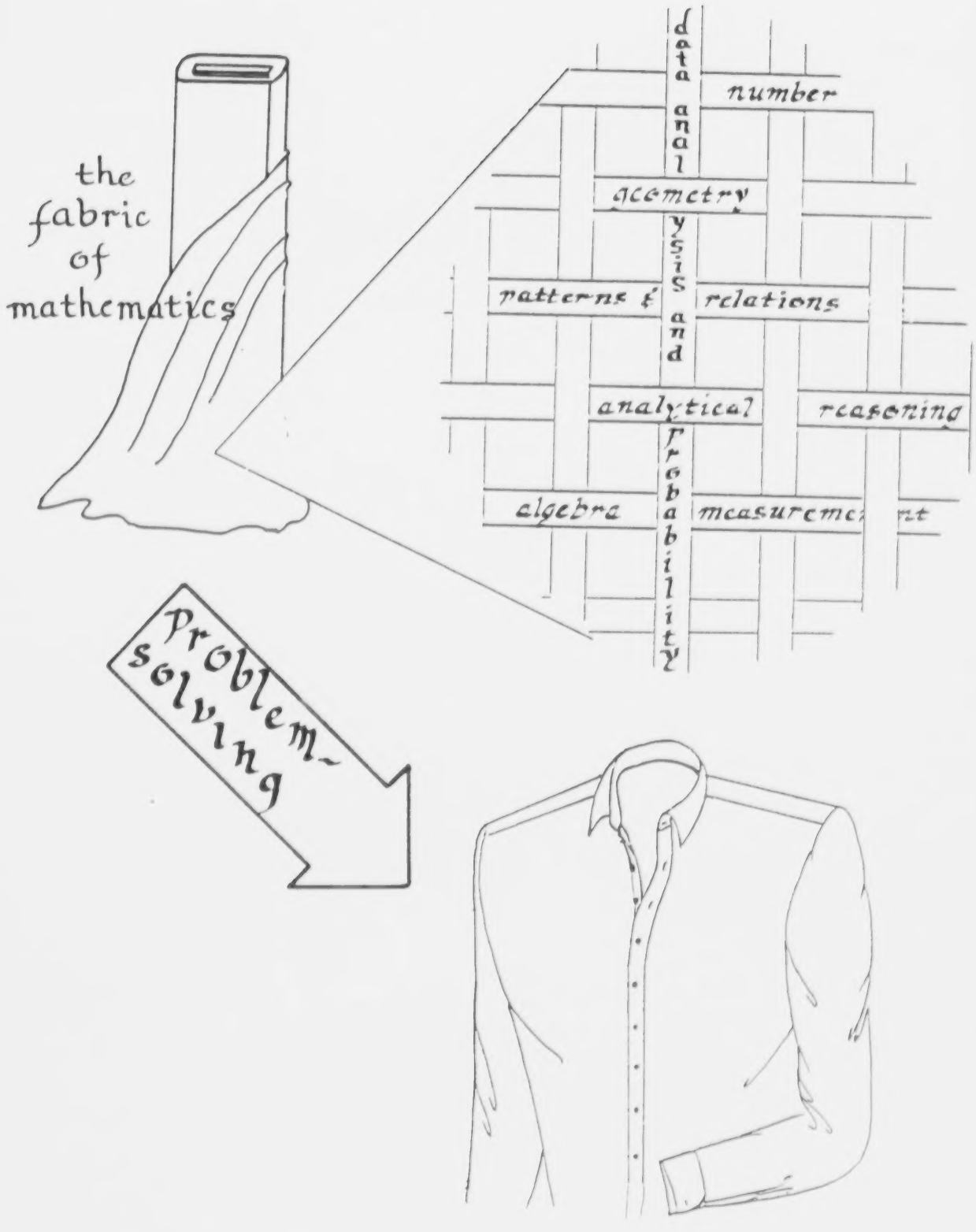
ACKNOWLEDGMENTS

The members of the Math Essential Skills Committee wish to thank Walter Denham, Director of the Office of Mathematics, Science and Health, California Department of Education for the support and encouragement received as they constructed this document.

They are especially grateful to the California Department of Education for permission to adapt items from the California Framework.

The Committee also appreciates the work of Ms. Dale Curtis of Tucson who devoted a substantial amount of time to generating statistical analysis of the data received from the districts. Her work helped the Committee members assess the district responses.

The Committee and the Arizona Department of Education wish to express their gratitude to the Phoenix law firm of Shimmel, Hill, Bishop & Gruender, P.C. which donated the word processing and administrative staff services used in preparation of the Committee's numerous draft revisions and the final Committee report.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ysis and
patterns & relations
analytical reasoning
probability
algebra
measurement
Problem-solving

FOUNDATION

STATEMENT

FOUNDATION STATEMENT

Mathematical competence is a national resource as well as a personal asset. Thus, students deserve an opportunity to participate in a curriculum which fosters their understanding of mathematics, encourages mathematical thinking, provides for career options, and nurtures an interest in the field. It is expected that all students in the State of Arizona will have appropriate experiences to develop their mathematical abilities as well as their enjoyment of and fascination with the subject. The implementation of this document will ensure every student's right to participate successfully in mathematics.

Society continues to express a valid concern that basic or essential skills be a significant part of the education of every student. The position expressed by the recommendations in this instrument is that the scope of essential skills must include those things which are essential to meaningful, productive citizenship in today's world and in the future. Narrow limitation of the essential skills in mathematics to the mechanics of numerical computation is insufficient. Mathematics instruction must provide for understanding, application, problem solving, enjoyment and a ready willingness to use mathematical ways of thinking.

It must be understood that the skills essential in one time and place may not be adequate for another. The need for new skills arises with the passage of time and as societal and technological changes emerge. The value of facility with paper and pencil computations has been diminished by technological developments such as the electronic calculator. Effective use of instructional time means that a reasonable balance among concept development, problem solving and computational facility must be achieved.

Rapid advances in technology force educators to reexamine the emphasis being placed on certain aspects of mathematics content and methodology. The National Council of Teachers of Mathematics publication, *An Agenda for Action: Recommendations for School Mathematics of the 1980s*, states:

> It is dangerous to assume that skills from one era will suffice for another. Skills are tools. Their importance rests in the needs of the times. Skills once considered essential become obsolete, and this is likely to increase in pace and scope as advances in technology revolutionize our individual, social and economic lives. Necessary new skills arise from the dimensions of the mathematics pertinent to an age of population explosion, space exploration, economic and fiscal complexity and microelectronic wonders. Time and space for including these new skills in the curriculum must be purchased by eliminating the obsolete. Professional knowledge of future trends, industrial, financial, engineering and scientific need, and the demands of daily life are all better arbiters of what is currently essential and what has become obsolete than our nostalgia as parents or teachers.

PROBLEM SOLVING

Problem solving must be the focus of mathematics instruction. Problem solving is not only applying mathematics to the real world but also recognizing the mathematical relationships which emerge from real world situations. Instruction should be structured so that a substantial number of problem-solving situations, both routine and non-routine, are presented to the students. Teachers must be aware of this focus so they can provide these opportunities.

Students need many opportunities to develop their problem-solving skills; they should participate in situations which require them to organize data, interpret information, and make decisions. Student interaction, such as discussion of possible solutions which enhances each student's thinking, is essential. Persistence in solving problems, not speed, should be the goal. Placing the emphasis on quick, correct responses does not help establish an environment that encourages the process of solving problems. It is useful for students to develop an interest in solving problems, the confidence to try new strategies and a willingness to risk being wrong.

CONCRETE LEARNING EXPERIENCES

Recent studies of human learning support the theory that students' mental images and abstract ideas are based on their previous experiences. Research has further shown that learners require the use of concrete materials to relate this previous knowledge and background to new concepts. This is especially important in the teaching of mathematics which, by its nature, is abstract. The use of manipulatives not only helps students understand mathematical concepts, but also enables them to apply these ideas to real-world situations.

Concrete materials are most commonly found in the primary grades. Although they are very important at this level, all learners benefit greatly from "hands-on" activities. At the middle school and secondary levels, concrete materials can take many different forms. Geometric models, diagrams, pictures and drawings can enhance the learning of many sophisticated concepts, particularly when a concept is new.

The classroom teacher plays a vital role in the effective use of concrete materials. Simply providing manipulatives in a classroom is not enough. The teacher must interact with the students and guide them to think mathematically. One of the roles of the teacher is to provide the language that will allow for the transition from manipulation of the materials to verbalization of the actions and, finally, internalization of the concepts. This is by no means a simple task, yet it is an essential one in the learning of mathematics.

THE LEARNING ENVIRONMENT

Meaningful learning comes from students' personal experiences with concepts and ideas. Teaching by telling does not work. Students must become familiar with the language of mathematics, organize their thinking and assess their understandings.

Teaching for understanding demands a classroom environment which offers chances for students to work in cooperative learning groups. This allows students to learn from each other as well as from the teacher. This supportive atmosphere encourages risk taking—a necessary element in the learning process.

Integrating the mathematics curriculum with other subject areas is vital for student understanding of how knowledge in various content areas is interrelated. This process allows students to personalize learning, making it more meaningful. Minimal integration may be achieved by indicating how and where various mathematical concepts may be used outside the classroom. Full-scale integration may be achieved by developing curriculum which combines subject matter areas.

It will also become necessary to re-evaluate some aspects of mathematics teaching and establish new priorities. The role of standardized testing in our evaluation of mathematics learning must be examined. Emphasis should be placed on teacher observation of students, testing for

understanding of concepts, and assessing application and problem-solving skills. The teaching of mathematics must focus on what students learn and not the amount of material covered.

TEACHING FOR UNDERSTANDING

The ability to apply or use knowledge is a worthwhile goal for mathematics instruction. Such instruction must provide activities which give students experience in application. It is equally evident that teaching must also be directed toward understanding. Algorithms or computational manipulations are based on mathematical principles. Students should acquire knowledge and understanding of those underlying principles. An understanding of mathematical concepts can be fostered by a judicious, time/cost-effective balance of "guided discovery" and "meaningful verbal" teaching. The efficacy of discovery in promoting understanding is well known, as is its cost in terms of classroom time. Verbal learning which avoids the hazards of rote learning provides an effective balance for expanding and continuing the results of discovery learning. Planning for the teaching of mathematics should provide for understanding and application as well as for simple transmission of knowledge.

For this program to be effective, each and every student must be instructed in each strand, not only from a computational standpoint but, more importantly, from a conceptual standpoint. New concepts need to be presented in such a way that all students can understand the basic ideas. Many students will go beyond the fundamentals in their study. Lower-achieving students, by learning these concepts, will have appropriate experiences in all of the strands. At no time should computational work, while important, overshadow conceptual understanding.

THE NATURE OF MATHEMATICS AND HOW IT SHOULD BE TAUGHT

Mathematics is a complex and broadly inclusive subject. It is possible to identify different areas or strands of the subject which foster understanding of its complexity and inclusiveness. It is important that each of these receives appropriate emphasis in school mathematics programs, and yet they must be understood as inseparable parts of mathematics. One piece at a time can be examined but not separated from the organic whole. The logical, sequential nature of mathematics strongly suggests certain orderings of the subject matter for the effective teaching of it. The strands identified in this document pervade mathematics, recurring with increasing complexity, sublety and power as students progress in their mathematical growth.

Appropriate consideration of the nature of mathematics itself, along with recognition of the intellectual growth of students, gives rise to the concept of the "spiral curriculum" in mathematics. In such a curricular organization, concepts are visited and revisited at various grade levels. The complexity of treatment and the level of discrimination required of students increases with successive revisitations as the students' intellectual development permits.

THE SIGNIFICANCE OF MATHEMATICS

The study of mathematics should be given a place of importance in the curriculum comparable to that of the study of language. Instruction in mathematics must demonstrate the power of the subject to students, as well as provide experiences rich in problem-solving applications appropriate to their levels of maturity. Nothing else can be so convincing of the value of mathematics to society as the students' perception of its value in their world.

It must be understood that mathematics provides structure, order, and predictability so that students can understand and effectively interact with both natural and social environments. Intellectual disciplines grow in credibility and acceptance in direct proportion to the extent to which they become mathematized. Appreciation of mathematics in our culture dictates that greater emphasis be given to mathematics instruction in elementary and secondary schools. Mathematical models for prediction and description can be applied to problem solving in the arts, humanities, and social sciences as well as the natural sciences.

STUDENT ATTITUDES

Goals for mathematics instruction include those related to attitudes and feelings as well as those related to skills and understandings. Mathematics anxiety inhibits the learning of many students, and mathematics teachers need to be aware of its causes and ways to lessen its effects. Instruction should include procedures which foster enjoyment of learning mathematics and appreciation of its great power. Students who grow up learning to enjoy mathematics become adults with a ready willingness to use mathematics to deal with daily problems and issues. As students, they are more inclined to continue their pursuit of mathematical studies voluntarily. They are much less likely to find pathways to future academic and career goals blocked by premature termination of formal study in mathematics. Teaching which encourages conjecture and intuitive approaches to problem solving seems to help students enjoy mathematics. Students who are encouraged to place as much value on thought processes as on answers are more apt to use mathematics enthusiastically than are those who focus mainly on correct answers.

ADDRESSING THE NEEDS OF A DIVERSE STUDENT POPULATION

Because of the nature of the student population in Arizona, mathematics teachers must be prepared to address diversity. Teachers need an awareness of research on learning styles and teaching methods that will provide a theoretical basis for the way the students' mathematics experiences are planned and organized. Knowledge of methods to assess students' level of mental maturity, degree of mathematics readiness and attitude toward the subject helps mathematics teachers design the most appropriate instruction.

This Committee feels the objectives set forth in this document effectively raise expectations for **ALL** students in the State of Arizona.

TECHNOLOGY

The use of technology in the classroom has not yet realized its full potential, this despite the widespread use of calculators and computers outside the school. If our schools and, more importantly, our students are to participate in these advances, then this technology must become an integral part of the school curriculum. The National Council of Teachers of Mathematics in their Position Statement on Calculators in the Mathematics Classroom "recommends the integration of the calculator into the school mathematics program at all levels in classwork, homework and evaluation."

Many people fear that the regular use of calculators will result in students' failure to learn basic number facts or inability to perform basic computations. Research has demonstrated

otherwise. Estimation and mental computation become extremely important when using a calculator, and both of these abilities require number skills.

Incorporating calculators into the curriculum allows students of all ability levels to experience and explore a broad range of concepts and understandings. No longer are solutions to problems unattainable because of unwieldy computations. With the use of calculators, students are able to focus on concepts, problem solving, and mathematical relationships without becoming bogged down with tedious calculations.

Computers, as well as calculators, have become an important feature of modern society. Schools have an exciting opportunity to take advantage of the capabilities of this technology. Using existing computer software, students can manipulate large amounts of data, perform calculations too complicated or time consuming for the calculator, and produce satisfying graphical representations of data. Students can develop and test problem-solving strategies by programming computers or by using programs which simulate real-world situations or present strategy games. As the hardware becomes more sophisticated and the software becomes interwoven with the existing curriculum, many new and exciting educational experiences can take place. Computers have the potential of providing learning experiences that previously were beyond the means of teachers, textbooks and supplementary materials.

STAFF DEVELOPMENT: PRESERVICE AND INSERVICE

Because the teacher is often the determining factor in developing positive student attitudes and interest in mathematics, it is essential that the legislature, universities and school districts provide major support for preservice and inservice education. Teachers must be supported in their efforts to relate the goals of this document to the specific needs of their students.

Prospective teachers of mathematics, both elementary and secondary, need a firm foundation in mathematics instruction so that they are aware of its importance in this technological age and prepared to continue their mathematical growth while they are teaching. Because of the specialized needs of teachers, their mathematical background will need to encompass a broad array of topics and areas of emphasis. The 1983 report of the Committee on the Undergraduate Program in Mathematics (CUPM), Mathematical Association of America, describes a program that fulfills this need. It is recommended that the levels and courses in that report serve as a model for teacher preparation in Arizona.

Prospective teachers must be concerned not only with mathematical concepts, but also with the means of communicating these concepts to their students. Teachers need adequate preparation in understanding how people learn and in the methodology appropriate for their needs. This preparation should include instruction in a variety of teaching techniques ensuring that teachers will have them at their disposal to use in different situations.

A special endorsement should be available from the state to elementary teachers who want to focus on mathematics. These mathematics specialists could not only support teachers but also enrich the mathematics program in their schools. Services they could provide include conducting mathematics workshops, furnishing resource materials to other teachers and serving as contact people within their schools. This endorsement is especially appealing because the cost would be minimal, yet the benefits reaped by the students and other teachers would be immense.

Inservice programs for teachers need to be ongoing. Teachers need opportunities to extend their knowledge of teaching techniques, to use these techniques in the classroom and to reflect on the changes. A society which hopes to have education as a lifelong goal for its citizens must support and encourage the efforts of its teachers to continue learning throughout their teaching careers. Teacher education initiated in universities must be continued through inservice.

MATHEMATICS PROGRAM

FOR PRIMARY STUDENTS

THE ESSENTIAL SKILLS IN MATHEMATICS

Mathematics Program for Primary Students

Young children come to school with a variety of mathematical experiences. The primary mathematics program should focus on developing concepts by relating mathematics to children's experiences. Labeling concepts with the appropriate name and symbol should always follow developing the concept using real things. These experiences provide opportunities for children to develop an understanding and appreciation of mathematics. These early years are important years during which children's confidence and attitude toward mathematics are established.

It is essential that children be provided a supportive classroom atmosphere in which exploration and investigation are encouraged and teachers provide guidance to help children discover mathematical ideas and their relationships. Because young children are not abstract thinkers, there must be frequent use of concrete materials and instruction to help children relate these experiences to their symbolic representations. Students need opportunities to interact with each other by talking and listening to each other's ideas. There must also be time to organize and interpret information.

Number

The goal of activities involving number should be to help students develop number sense. Instruction should emphasize understanding the procedures. Many experiences with concrete materials are necessary to develop an understanding of place value, the foundation for much of the later work with numbers. Estimation must be a part of the primary mathematics program. This includes estimations of real quantities as well as approximations of answers before actual calculations. These estimations should be built upon concrete experiences.

Measurement

Students should experience informal comparisons. Early experiences with nonstandard units of measure will develop into the need for standardized units. Many opportunities to estimate measures of length, weight and time are necessary to the notion that measure is approximate.

Geometry

Geometric experiences for primary students should be informal, using the environment to develop language and vocabulary. Many experiences with concrete materials help students see relationships and develop insights into the patterns of geometry.

Pattern

Pattern is the basis of all mathematics. The study and understanding of pattern can have a positive influence on children's abilities to understand mathematics. Children who do not see patterns, often do not expect things to make sense; they may see all events as discrete, separate and unrelated. Children who have not been taught to look for patterns might write the numbers from 1 to 100 as 100 separate tasks. Such children will frequently get confused and rarely recognize or correct mistakes. Young children, therefore, should be encouraged to discover, describe and represent patterns.

Data Analysis

Students can begin their study of data analysis by constructing and interpreting graphs. For young children these activities would include constructing real graphs, talking about the information on graphs, and creating picture and symbolic graphs to represent the data. These discussions lay a foundation for solving meaningful word problems by demonstrating a useful problem-solving tool.

Analytical Reasoning

Logical thinking and problem solving should be the focus of mathematics programs. Situational problems encourage students to use what they know in new situations (How many crackers will we need for our group if each person gets two?). As students understand basic concepts and learn the basic arithmetic operations, they should be taught how and when to use the calculator and computer. Students should gain the knowledge and confidence necessary to develop an appreciation of the beauty of mathematics.

K-3 Outcomes

Mathematics instruction in Grades K-3 should ensure that students completing Grade 3 have experiences which enable them to perform the following outcomes:

I. **Number**

1. Count by ones, twos, fives and tens.
2. Use cardinal and ordinal numbers to compare and order quantities.
3. Demonstrate an understanding of the meaning of the four basic operations.
4. Have facility with basic addition, subtraction and multiplication facts.
5. Use concrete materials or models to demonstrate an understanding of place value.
6. Explore the concepts of multiplication and division with concrete materials.
7. Read and write the number represented when objects are grouped by hundreds, tens and ones.
8. Estimate answers to computational problems.
9. Add or subtract two three-digit whole numbers.
10. Choose the appropriate operation to be used in a given situation.
11. Interpret word problems by using role playing, pictures and models.
12. Write mathematical sentences to represent a situation.
13. Use informally the properties of commutativity, associativity and identity.

14. Use concrete materials to recognize, represent and compare halves, thirds and fourths.

15. Recognize fractional equivalents for halves, fourths and tenths.

16. Recognize and count money.

17. Use money to represent and compare decimal values.

II. Measurement

1. Use nonstandard, metric and English units of measure to estimate and measure length, volume and weight.

2. Use digital and conventional clocks to tell time.

3. Read and interpret Celsius and Fahrenheit temperatures on thermometers.

4. Choose an appropriate unit of measure in a given situation.

5. Use a variety of measurement instruments.

III. Geometry

1. Use terminology appropriate to the grade level.

2. Use visual attributes, concrete materials and appropriate vocabulary to identify, classify and describe common geometric figures and models.

3. Use several geometric shapes to make other geometric shapes.

4. Decide whether figures are congruent and whether they are similar.

5. Explore the filling of space using manipulatives.

6. Use manipulative materials to develop the concepts of point, line and line segment.

IV. Patterns and Relations

1. Identify, describe and extend a pattern in a sequence of objects.

2. Use a concrete model to create a pattern and represent that pattern symbolically.

3. Describe the relationship given in a table of numbers derived from a sequence of objects.

4. Determine a location by using ordered pairs of numbers on a rectangular grid.

V. Data Analysis and Probability

1. Collect, organize, represent and interpret data derived from surveys and experiments conducted by the students.

2. Create and interpret concrete, pictorial and symbolic graphs.

3. Perform simple activities involving probability.

VI. Analytical Reasoning

1. Classify and sort objects by observing relationships and making generalizations.

2. Make reasonable or logical conjectures and conclusions about situations with concrete materials, using such words as these: *and, or, if . . . then, all, some, none, not* and *out of*.

MATHEMATICS PROGRAM

FOR GRADES 4-8

Mathematics Program For Grades 4-8

Students in grades four through eight are developing lifelong attitudes toward mathematics while they expand their knowledge of the field. Thus, teachers must take great care in the formation of these attitudes so that young people not only will use mathematics, but will enjoy it and desire to continue their study of it. The potential for student interest and motivation is great; it must not be lost.

Solving routine and non-routine problems is one means of creating interest in the middle school classroom. Problems should be utilized to introduce concepts, to provide applications for skills and concepts, and to promote the integration of the strands delineated in this document. By this age, students have learned enough mathematics that they can be given ample opportunities to see how this knowledge is a part of real-world situations. They must realize that the mathematics they know can solve problems and also can be used to enrich their view of the world.

In addition to applying concepts already learned, students need to strengthen and expand understandings and skills acquired in earlier grades. New concepts must be learned in order to prepare students for study of mathematics in high school. At the same time, students need to begin to think independently, as well as to work cooperatively in small groups while discovering ideas and solving problems. It must not be forgotten that concrete materials, calculators, computers and other aids are an integral part of the implementation of the total curriculum.

It is essential that instruction in these grades promotes higher-level thinking skills. These students are of the age when they can begin to think formally and abstractly. They must be given the chance to do so in a supportive, inquisitive and mathematically rich environment.

4-8 Outcomes

Mathematics instruction in grades 4-8 should ensure that students completing grade 8 have experiences which enable them to perform the following outcomes:

I. Number

1. Use concrete materials and illustrative models to represent place value in whole and decimal numbers, fractions and their equivalents and percents.
2. Read and write whole numbers, integers, common fractions and decimal fractions.
3. Use appropriate symbols and order relations when comparing whole numbers, integers, common fractions and decimal fractions.
4. Identify and use number properties to simplify expressions and calculations: *i.e.* commutative, associative, distributive, identity of one and zero, rule of order of operations.
5. Convert numbers in base 10 notation to and from scientific notation.
6. Convert numbers in base 10 notation to and from expanded notation.
7. Understand the square root of a number and the methods of finding it or its approximation.

8. List the prime factorization of a whole number and ultimately write it as a numerical expression involving positive exponents.

9. Understand greatest common factor and least common multiple and their applications.

10. Find equivalent fractions.

11. Recall addition, subtraction, multiplication and division facts with reasonable speed.

12. Use basic operations with whole numbers, integers, common fractions and decimal fractions.

13. Use various techniques of mental arithmetic.

14. Estimate to predict answers to computational problems and to check for reasonableness of answers.

15. Use a calculator in appropriate situations.

16. Use the additive and multiplicative laws of exponents.

17. Use ratios and proportions to solve problems.

18. Convert among percent, fractional and decimal equivalents.

19. Solve for the unknown in a percent problem.

20. Interpret problems, translate to numerical expressions, choose an appropriate method of calculation and solve.

21. Solve problems involving money.

22. Round off numbers to a specified place value.

II. Measurement

1. Know that all measurement is approximate.

2. Select the appropriate measuring instrument.

3. Add and subtract measures of time.

4. Compare temperatures and determine the amount of change.

5. Measure length, volume and weight in both English and metric units.

6. Convert English and metric measurement units to equivalent units within the given system.

7. Be aware of approximate equivalent measurements between English and metric units.

8. Use an appropriate formula to calculate the perimeter and area of polygons and the circumference of a circle.

9. Use an appropriate formula to calculate the volume and surface area of the following solids: spheres, prisms, pyramids, cylinders and cones.

10. Use a protractor to measure and draw angles.

11. Use an appropriate formula to determine a measurement when a direct measurement tool is unavailable.

12. Estimate lengths, areas, volumes and weights, and check by measuring in English and metric units.

13. Solve problems with measurements including distance, weight, time, area, capacity and temperature.

III. Geometry

1. Use terminology appropriate to the grade level.

2. Recognize two and three dimensional figures by their shapes and identify their component parts and associated properties.

3. Through observation, measurement, drawing and modeling, identify geometric properties such as symmetry, congruence, similarity, parallelism and perpendicularity.

4. Visualize, draw and construct two and three dimensional figures through concrete experiences.

5. Utilize geometric principles and ideas to solve problems.

6. Experiment with, discover and apply the known formulas of geometry.

7. Understand pi and its importance in the geometry of the circle.

8. Recognize and classify angles as well as identify relationships between angles.

9. Draw and/or construct a variety of shapes having the same area.

IV. Patterns and Relations

1. Organize data to show relationships in tables and graphs.

2. Interpret data from tables and charts to determine relationships.

3. Determine an expression or an equation for a relationship and then evaluate or solve it for a given value of the variable.

4. Recognize or find a specific pattern occurring in a sequence of numbers.

5. Extend patterns and create new ones.

6. Graph ordered pairs to show a pattern or relationship.

7. Understand the relationship between angles that are complementary, supplementary and vertical.

8. Identify and understand the relationships of angles formed by a transversal of two parallel lines.

9. Understand the relationships occurring between similar figures.

V. Data Analysis and Probability

1. Collect and organize data using lists, tables and graphs.

2. Interpret, analyze, and draw conclusions from graphs.

3. Take polls and surveys.

4. Predict and record the probability of events from simple experiments.

5. Express probability as a ratio or fraction.

6. Find the mean, median, mode and range for a set of data.

7. Determine the probability of simple events and draw conclusions or make interpretations.

8. Find the empirical probability of an event from a sample of observed outcomes.

9. Find the probability of complementary events and of mutually exclusive events.

10. Generate a frequency distribution for a given list of data.

11. Use a list or tree diagram to count permutations or combinations.

VI. Analytical Reasoning

1. Recognize and identify numerical and geometric patterns and sequences.

2. Classify and categorize sets of numbers or groups of geometric shapes.

3. Using a given attribute, find similarities and differences among geometric shapes or designs.

4. Appropriately use terms such as *and, or, not, only,* and *if . . . then,* in a mathematical sense.

5. Provide specific examples of a given numerical or geometric principle.

6. Provide a counterexample of a condition that is not always true.

7. Perform tasks involving inductive and deductive reasoning.

VII. Algebra

1. Understand the meaning of "variable" and "constant."

2. Be familiar and proficient with the standard order of operations.

3. Represent mathematical relationships using variables.

4. Simplify polynomial expressions.

5. Evaluate simple algebraic expressions by substituting values for the variables.

6. Solve two-step equations and inequalities using integers, fractions and decimals.

MATHEMATICS PROGRAM

FOR GRADES 9-12

Mathematics Program For Grades 9-12

The Committee's recommendations for the mathematics curriculum in grades 9-12 have been organized into two sections. The first section lists the core of skills expected of all high school graduates. The second section describes additional courses and skills which students need in order to comply with the Regents' entrance requirements to Arizona's state universities.

Providing Alternative Opportunities

The diversity of interests and ability levels of high school students requires the existence of a variety of mathematical learning opportunities. The Committee hopes that districts will consider providing many avenues to study mathematics. All students should be encouraged to learn and use mathematics throughout their high school years and, thus, find it a useful thinking mode throughout life, regardless of their career choices.

Courses to supplement the Regents' requirements might include the following:

Trigonometry
Precalculus
Calculus
Discrete Mathematics
Probability
Statistics

Traditional review and remedial mathematics courses stressing drill or practice in manipulative/computation activities isolated from realistic, credible applications are ineffective for a significant part of the student population. Schools are encouraged to provide mathematics courses which relate to various career or vocational clusters, such as construction, industrial arts and shop, business and finance, agriculture, homemaking and computers.

Adjusting the Course of Study for All High School Students

The two-credit requirement in R-7-302 is insufficient for the goal of student mastery of the Essential Skills for all high school students if districts interpret the two-credit requirement as two classes of basic math. The Essential Skills take students beyond basic math, and high school courses of study should be adjusted accordingly. The high school mathematics program should make it possible for students to take mathematics during all four years and encourage students to participate in Regent-required classes when ready.

Mathematical Competencies Expected of All High School Graduates

High school mathematics instruction should ensure that each student graduating from high school performs the following outcomes:

I. Number

1. Use the four arithmetic operations on rational numbers (expressed as common fractions, decimal fractions or percents) with accuracy and reasonable speed, using pencil and paper or calculator.

2. Perform mental calculations of the four operations with appropriate levels of precision.

3. Understand the concept of ordering the real numbers by locating their corresponding points on the number line.

4. Write mathematical expressions and sentences to replace verbal expressions, using appropriate operation, number and relation symbols.

5. Use standard rules for order of operations together with symbols of inclusion to correctly evaluate numerical expressions and formulas in a variety of problem settings.

6. Understand and use properties of equality and inequality.

II. Measurement

1. Understand the approximate nature of measurement and apply the precision of measurements to the resulting accuracy of calculations.

2. Make unit conversions with appropriate levels of accuracy between and within standard systems of measures.

3. Use appropriate standard units of measurement to make reasonable estimates of linear, area, volume and weight measures of objects commonly encountered in daily life.

4. Select and use appropriate formulas or procedures to indirectly determine a measure when a direct measurement is not available or feasible.

III. Geometry

1. Identify and distinguish between geometric elements, such as lines, angles, planes, polygons, circles, and regular solids.

2. Understand and apply basic geometric relationships such as parallel, intersecting, perpendicular, similarity, proportionality, and congruence.

3. Calculate areas of regular polygons and circles and volumes of rectangular solids, cylinders and spheres.

4. Use compass, protractor, and ruler to perform standard geometric constructions.

5. Apply the Pythagorean theorem in solving problems.

IV. Data Analysis and Probability

1. Select and use appropriate principles of counting collections and arrangements of objects or outcomes of sequential procedures.

2. Select and use appropriate statistical measures to describe sets of data.

3. Explain variability of statistical measures in terms of sampling process or population differences.

4. Use the concept of mathematical expectation to estimate outcomes of random processes.

5. Identify and explain misuses of statistics.

6. Use experimental observations to estimate empirical probabilities and population parameters.

7. Distinguish between independent and dependent events and use conditional probabilities.

V. Algebra

1. Add, subtract, multiply and divide with monomials and binomials.

2. Simplify and evaluate algebraic expressions which involve integral exponents and square roots.

3. Simplify rational algebraic expressions having monomial denominators.

4. Solve linear equations and inequalities in one variable, and apply these to solve problems.

5. Solve systems of linear equations and inequalities in two variables, graphically and analytically, and formulate such systems to solve problems.

6. Solve problems using ratio and proportion, percent or direct and inverse variation.

VI. Patterns and Relations

1. Identify and distinguish between arithmetic and geometric progressions and state a rule or formula for the general term of such a progression.

2. Write the linear relationship between two variables by inspection of its graph or of the set of its ordered pairs.

3. Graph the inverse of a function by inspection of the graph of the function.

4. Identify and explain graphic misrepresentation or distortions of sets of data.

5. Identify, interpret and construct graphs of nonlinear relations such as parabolas and circles.

VII. Analytic Reasoning

1. Distinguish between inductive and deductive reasoning and explain how each is most appropriately used.

2. Recognize when conditions of definitions are satisfied.

3. Use deductive reasoning to generate conclusions.

4. Use statistical reasoning to test hypotheses informally.

5. Identify valid and invalid arguments.

Additional Courses and Skills Required for University Admission

Upon completing courses in Algebra I, Algebra II, and Geometry, a high school student should have mastered the following outcomes:

I. Algebra I

1. Simplify algebraic expressions.

2. Evaluate algebraic expressions.

3. Solve linear equations.

4. Use the laws of exponents to simplify expressions involving integral exponents.

5. Add, subtract, multiply and divide polynomial expressions.

6. Factor polynomials.

7. Solve linear inequalities and graph them on a number line.

8. Find the slope of a line and write equations of lines in standard and slope intercept form.

9. Solve systems of linear equations in two variables by substitution, addition method and graphically.

10. Add, subtract, multiply and divide algebraic fractions and complex fractions.

11. Simplify, add, subtract, and multiply radicals, and rationalize denominators.

12. Solve direct and inverse variation problems.

13. Solve elementary word problems.

14. Use formulas.

15. Solve and graph absolute value problems.

16. Solve quadratic equations.

17. Understand the concept of functions and their graphs.

II. Algebra II

1. Evaluate algebraic expressions.

2. Use laws of exponents to change the forms of expressions involving integral exponents and rational exponents.

3. Simplify radicals.

4. Change from fractional exponents to radical form or radical form to fractional exponents whenever necessary.

5. Solve formulas for specified symbols.

6. Solve equations and inequalities involving absolute value.

7. Perform operations with polynomials including synthetic division.

8. Factor various types of polynomials.

9. Solve quadratic equations by factoring and using the quadratic formula.

10. Solve quadratic inequalities.

11. Perform operations with rational expressions.

12. Solve equations and inequalities involving rational expressions.

13. Perform operations with complex numbers.

14. Work with forms of the equation of a line and the concept of the slope of a line.

15. Solve linear systems of equations in three variables.

16. Work with the function concept and function notation.

17. Use the method of completing the square to find the vertex of a parabola and then graph.

18. Solve exponential and logarithmic equations.

19. Work with arithmetic and geometric progressions.

20. Use a calculator to obtain two-decimal-place approximations to the answer for radical, exponential or logarithmic forms.

21. Work with conic sections, and use algebra to solve problems involving coordinate geometry.

22. Solve equations and problems which rely on properties of logarithmic and exponential functions.

23. Understand degree and radian measure of angles.

24. Work with trigonometric functions as ratios of sides of right triangles and as circular functions.

25. Graph trigonometric functions.

26. Use calculators and tables to approximate values of trigonometric functions.

27. Derive exact values of trigonometric functions associated with angles of 30, 45, 60 and 90 degrees.

28. Describe and graph one-to-one functions and their inverses.

29. Work with inverse trigonometric functions.

30. Verify trigonometric identities and solve trigonometric equations.

31. Solve right triangles and use the law of sines and the law of cosines.

III. Geometry

1. Use definitions, postulates, and theorems to develop proofs.

2. Solve word problems.

3. Perform algebraic manipulations.

4. Use formal, logical reasoning to solve abstract, theoretical and practical problems.

5. Understand angle and line relationships, *e.g.*, vertical angles, parallel or perpendicular lines, and supplementary and complementary angles.

6. Use concepts of congruence and similarity.

7. Explore polygonal relationships to include and identify common geometric shapes and describe their angle measures.

8. Explore circles and arc characteristics to determine the measure of arcs and their related angles and chords, tangents and secants.

9. Know and use the Pythagorean theorem.

10. Find the perimeter and area of a plane figure.

11. Find volume and surface area of a solid.

12. Make and apply geometric construction.

13. Solve problems applying the methods of coordinate geometry, *e.g.*, the distance formula, the midpoint formula, and finding the equation of a line.

14. Use right triangle trigonometry to solve problems.

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